DEFINITIONS, PROPERTIES, POSTULATES, and THEOREMS

Definitions

- **Congruent Segments** – Segments have equal length if and only if they are congruent.
- **Congruent Angles** – Angles have equal measure if and only if they are congruent.
- **Midpoint** – A point is a midpoint if and only if it divides a segment into two congruent segments.
- **Segment Bisector** – A segment, ray, line, or plane is a segment bisector if and only if it divides a segment into two congruent segments.
- **Angle Bisector** – A ray is an angle bisector if and only if it divides an angle into two congruent angles.
- **Complementary angles** – Two angles are complementary angles if and only if the sum of their measures is 90°.
- **Supplementary angles** – Two angles are supplementary angles if and only if the sum of their measures is 180°.
- **Perpendicular lines** – Two lines are perpendicular lines if and only if they intersect to form a right angle.
- **Right angle** – An angle is a right angle if and only if it measures 90°.

PROPERTIES OF EQUALITY AND CONGRUENCE

Reflexive Property

Equality $AB = AB$
$m\angle A = m\angle A$

Congruence $\overline{AB} \cong \overline{AB}$
$\angle A \cong \angle A$

Symmetric Property

Equality
If $AB = CD$, then $CD = AB$.
If $m\angle A = m\angle B$, then $m\angle B = m\angle A$.

Congruence
If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.
If $\angle A \cong \angle B$, then $\angle B \cong \angle A$.

Transitive Property

Equality
If $AB = CD$ and $CD = EF$, then $AB = EF$.
If $m\angle A = m\angle B$ and $m\angle B = m\angle C$, then $m\angle A = m\angle C$.

Congruence
If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.
If $\angle A \cong \angle B$ and $\angle B \cong \angle C$, then $\angle A \cong \angle C$.

PROPERTIES OF EQUALITY

Addition Property

Adding the same number to each side of an equation produces an equivalent equation.

Example
$x - 3 = 7$
$x - 3 + 3 = 7 + 3$

Subtraction Property

Subtracting the same number from each side of an equation produces an equivalent equation.

Example
$y + 5 = 11$
$y + 5 - 5 = 11 - 5$

Multiplication Property

Multiplying each side of an equation by the same nonzero number produces an equivalent equation.

Example
$\frac{1}{4}z = 6$
$\frac{1}{4}z \cdot 4 = 6 \cdot 4$

Division Property

Dividing each side of an equation by the same nonzero number produces an equivalent equation.

Example
$8x = 16$
$\frac{8x}{8} = \frac{16}{8}$

Substitution Property

Substituting a number for a variable in an equation produces an equivalent equation.

Example
$x = 7$
$2x + 4 = 2(7) + 4$

Postulates

1 Two Points Determine a Line
Through any two points there is exactly one line. *(p. 14)*

2 Three Points Determine a Plane
Through any three points not on a line there is exactly one plane. *(p. 14)*

3 Intersection of Two Lines
If two lines intersect, then their intersection is a point. *(p. 22)*

4 Intersection of Two Planes
If two planes intersect, then their intersection is a line. *(p. 22)*

5 Segment Addition Postulate
If B is between A and C, then $AC = AB + BC$.
If $AC = AB + BC$, then B is between A and C. *(p. 29)*

6 Angle Addition Postulate
If P is in the interior of $\angle RST$, then the measure of $\angle RST$ is the sum of the measures of $\angle RSP$ and $\angle PST$. *(p. 37)*

7 Linear Pair Postulate
If two angles form a linear pair, then they are supplementary. *(p. 75)*

8 Corresponding Angles Postulate
If two parallel lines are cut by a transversal, then corresponding angles are congruent. *(p. 128)*

9 Corresponding Angles Converse
If two lines are cut by a transversal so that corresponding angles are congruent, then the lines are parallel. *(p. 137)*

10 Parallel Postulate
If there is a line and a point not on the line, then there is exactly one line through the point parallel to the given line. *(p. 144)*

11 Perpendicular Postulate
If there is a line and a point not on the line, then there is exactly one line through the point perpendicular to the given line. *(p. 144)*

12 Side-Side-Side Congruence Postulate (SSS)
If three sides of one triangle are congruent to three sides of a second triangle, then the two triangles are congruent. *(p. 241)*

13 Side-Angle-Side Congruence Postulate (SAS)
If two sides and the included angle of one triangle are congruent to two sides and the included angle of a second triangle, then the two triangles are congruent. *(p. 242)*

14 Angle-Side-Angle Congruence Postulate (ASA)
If two angles and the included side of one triangle are congruent to two angles and the included side of a second triangle, then the two triangles are congruent. *(p. 250)*

15 Angle-Angle Similarity Postulate (AA)
If two angles of one triangle are congruent to two angles of another triangle, then the two triangles are similar. *(p. 372)*

16 Arc Addition Postulate
The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs. *(p. 602)*

Theorems

2.1 Congruent Complements Theorem
If two angles are complementary to the same angle, then they are congruent. *(p. 69)*

2.2 Congruent Supplements Theorem
If two angles are supplementary to the same angle, then they are congruent. *(p. 69)*

2.3 Vertical Angles Theorem
Vertical angles are congruent. *(p. 76)*

3.1
All right angles are congruent. *(p. 114)*

3.2
If two lines are perpendicular, then they intersect to form four right angles. *(p. 114)*

3.3
If two lines intersect to form adjacent congruent angles, then the lines are perpendicular. *(p. 115)*

3.4
If two sides of adjacent acute angles are perpendicular, then the angles are complementary. *(p. 115)*

3.5 Alternate Interior Angles Theorem
If two parallel lines are cut by a transversal, then alternate interior angles are congruent. *(p. 129)*

3.6 Alternate Exterior Angles Theorem
If two parallel lines are cut by a transversal, then alternate exterior angles are congruent. *(p. 130)*

3.7 Same-Side Interior Angles Theorem
If two parallel lines are cut by a transversal, then same-side interior angles are supplementary. *(p. 131)*

3.8 Alternate Interior Angles Converse
If two lines are cut by a transversal so that alternate interior angles are congruent, then the lines are parallel. *(p. 138)*

3.9 Alternate Exterior Angles Converse
If two lines are cut by a transversal so that alternate exterior angles are congruent, then the lines are parallel. *(p. 138)*

3.10 Same-Side Interior Angles Converse
If two lines are cut by a transversal so that same-side interior angles are supplementary, then the lines are parallel. *(p. 138)*

3.11
If two lines are parallel to the same line, then they are parallel to each other. *(p. 145)*

3.12
In a plane, if two lines are perpendicular to the same line, then they are parallel to each other. *(p. 145)*

4.1 Triangle Sum Theorem
The sum of the measures of the angles of a triangle is 180°. *(p. 179)*

Corollary to the Triangle Sum Theorem
The acute angles of a right triangle are complementary. *(p. 180)*

4.2 Exterior Angle Theorem
The measure of an exterior angle of a triangle is equal to the sum of the measures of the two nonadjacent interior angles. *(p. 181)*

4.3 Base Angles Theorem
If two sides of a triangle are congruent, then the angles opposite them are congruent. *(p. 185)*

4.4 Converse of the Base Angles Theorem
If two angles of a triangle are congruent, then the sides opposite them are congruent. *(p. 186)*

4.5 Equilateral Theorem
If a triangle is equilateral, then it is equiangular. *(p. 187)*

4.6 Equiangular Theorem
If a triangle is equiangular, then it is equilateral. *(p. 187)*

4.7 Pythagorean Theorem
In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs. *(p. 192)*

4.8 Converse of the Pythagorean Theorem
If the square of the length of the longest side of a triangle is equal to the sum of the squares of the lengths of the other two sides, then the triangle is a right triangle. *(p. 200)*

4.9 Intersection of Medians of a Triangle
The medians of a triangle intersect at the centroid, a point that is two thirds of the distance from each vertex to the midpoint of the opposite side. *(p. 208)*

4.10
If one side of a triangle is longer than another side, then the angle opposite the longer side is larger than the angle opposite the shorter side. *(p. 212)*

4.11
If one angle of a triangle is larger than another angle, then the side opposite the larger angle is longer than the side opposite the smaller angle. *(p. 212)*

4.12 Triangle Inequality
The sum of the lengths of any two sides of a triangle is greater than the length of the third side. *(p. 213)*

5.1 Angle-Angle-Side Congruence Theorem (AAS)
If two angles and a non-included side of one triangle are congruent to two angles and the corresponding non-included side of a second triangle, then the two triangles are congruent. *(p. 251)*

5.2 Hypotenuse-Leg Congruence Theorem (HL)
If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and a leg of a second right triangle, then the two triangles are congruent. *(p. 257)*

5.3 Angle Bisector Theorem
If a point is on the bisector of an angle, then it is equidistant from the two sides of the angle. *(p. 273)*

5.4 Perpendicular Bisector Theorem
If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment. *(p. 274)*

6.1 Quadrilateral Interior Angles Theorem
The sum of the measures of the interior angles of a quadrilateral is 360°. *(p. 305)*

6.2
If a quadrilateral is a parallelogram, then its opposite sides are congruent. *(p. 310)*

6.3
If a quadrilateral is a parallelogram, then its opposite angles are congruent. *(p. 311)*

6.4
If a quadrilateral is a parallelogram, then its consecutive angles are supplementary. *(p. 311)*

6.5
If a quadrilateral is a parallelogram, then its diagonals bisect each other. *(p. 312)*

6.6
If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram. *(p. 316)*

6.7
If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram. *(p. 316)*

6.8
If an angle of a quadrilateral is supplementary to both of its consecutive angles, then the quadrilateral is a parallelogram. *(p. 317)*

6.9
If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram. *(p. 318)*

Rhombus Corollary
If a quadrilateral has four congruent sides, then it is a rhombus. *(p. 326)*

Rectangle Corollary
If a quadrilateral has four right angles, then it is a rectangle. *(p. 326)*

Square Corollary
If a quadrilateral has four congruent sides and four right angles, then it is a square. *(p. 326)*

6.10
The diagonals of a rhombus are perpendicular. *(p. 327)*

6.11
The diagonals of a rectangle are congruent. *(p. 327)*

6.12
If a trapezoid is isosceles, then each pair of base angles is congruent. *(p. 332)*

6.13
If a trapezoid has a pair of congruent base angles, then it is isosceles. *(p. 332)*

7.1 Perimeters of Similar Polygons
If two polygons are similar, then the ratio of their perimeters is equal to the ratio of their corresponding side lengths. *(p. 368)*

7.2 Side-Side-Side Similarity Theorem (SSS)
If the corresponding sides of two triangles are proportional, then the triangles are similar. *(p. 379)*

7.3 Side-Angle-Side Similarity Theorem (SAS)
If an angle of one triangle is congruent to an angle of a second triangle and the lengths of the sides that include these angles are proportional, then the triangles are similar. *(p. 380)*

7.4 Triangle Proportionality Theorem
If a line parallel to one side of a triangle intersects the other two sides, then it divides the two sides proportionally. *(p. 386)*

7.5 Converse of the Triangle Proportionality Theorem
If a line divides two sides of a triangle proportionally, then it is parallel to the third side. *(p. 388)*

7.6 Midsegment Theorem
The segment connecting the midpoints of two sides of a triangle is parallel to the third side and is half as long. *(p. 389)*

8.1 Polygon Interior Angles Theorem
The sum of the measures of the interior angles of a convex polygon with n sides is $(n - 2) \cdot 180°$. *(p. 417)*

8.2 Polygon Exterior Angles Theorem
The sum of the measures of the exterior angles of a convex polygon, one angle at each vertex, is 360°. *(p. 419)*

8.3 Areas of Similar Polygons
If two polygons are similar with a scale factor of $\frac{a}{b}$, then the ratio of their areas is $\frac{a^2}{b^2}$. *(p. 433)*

10.1 45°-45°-90° Triangle Theorem
In a 45°-45°-90° triangle, the length of the hypotenuse is the length of a leg times $\sqrt{2}$. *(p. 542)*

10.2 30°-60°-90° Triangle Theorem
In a 30°-60°-90° triangle, the hypotenuse is twice as long as the shorter leg, and the longer leg is the length of the shorter leg times $\sqrt{3}$. *(p. 549)*

11.1
If a line is tangent to a circle, then it is perpendicular to the radius drawn at the point of tangency. *(p. 595)*

11.2
In a plane, if a line is perpendicular to a radius of a circle at its endpoint on the circle, then the line is tangent to the circle. *(p. 595)*

11.3
If two segments from the same point outside a circle are tangent to the circle, then they are congruent. *(p. 597)*

11.4
If a diameter of a circle is perpendicular to a chord, then the diameter bisects the chord and its arc. *(p. 608)*

11.5
If one chord is a perpendicular bisector of another chord, then the first chord is a diameter. *(p. 609)*

11.6
In the same circle, or in congruent circles:

If two chords are congruent, then their corresponding minor arcs are congruent.

If two minor arcs are congruent, then their corresponding chords are congruent. *(p. 609)*

11.7 Measure of an Inscribed Angle
If an angle is inscribed in a circle, then its measure is half the measure of its intercepted arc. *(p. 614)*

11.8
If a triangle inscribed in a circle is a right triangle, then the hypotenuse is a diameter of the circle.

If a side of a triangle inscribed in a circle is a diameter of the circle, then the triangle is a right triangle. *(p. 615)*

11.9
If a quadrilateral can be inscribed in a circle, then its opposite angles are supplementary.

If the opposite angles of a quadrilateral are supplementary, then the quadrilateral can be inscribed in a circle. *(p. 616)*

11.10
If two chords intersect inside a circle, then the measure of each angle formed is one half the sum of the measures of the arcs intercepted by the angle and its vertical angle. *(p. 620)*

11.11
If two chords intersect inside a circle, then the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord. *(p. 622)*